Math Virtual Learning

Calculus AB

L'Hopital's Rule

May 1, 2020

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Lesson: May 1, 2020

Objective/Learning Target:
Lesson 5 Limits Review
Students will evaluate limits using L'Hopital's Rule.

Warm-Up:

Note: This is a review. For more examples refer back to your notes.

Watch Video: L'Hopital's Rule

Read Article: L’Hopital’s Rule

Notes:

L'Hopital's Rule

Suppose that we have one of the following cases,

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{0}{0} \qquad \text{OR} \qquad \lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\pm\infty}{\pm\infty}$$

where a can be any real number, infinity or negative infinity. In these cases we have,

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

Remember: You can only use L'Hopital's Rule if you have one of these indeterminate cases.

Examples:

Find the limit $\lim\limits_{x\to 2} \frac{\sqrt{7+x}-3}{x-2}$.

Solution.

Because direct substitution leads to an indeterminate form $\frac{0}{0}$, we can use L'Hopital's rule:

$$\lim_{x\to 2} \frac{\sqrt{7+x}-3}{x-2} = \left[\frac{0}{0}\right] = \lim_{x\to 2} \frac{\left(\sqrt{7+x}-3\right)'}{(x-2)'} = \lim_{x\to 2} \frac{\frac{1}{2\sqrt{7+x}}}{1}$$

$$= \frac{1}{2} \lim_{x\to 2} \frac{1}{\sqrt{7+x}} = \frac{1}{2} \cdot \frac{1}{3} = \frac{1}{6}$$

Examples:

Example 2 Evaluate the following limit.

$$\lim_{x \to 0^+} x \ln x$$

Hide Solution ▾

Note that we really do need to do the right-hand limit here. We know that the natural logarithm is only defined for positive x and so this is the only limit that makes any sense.

Now, in the limit, we get the indeterminate form $(0)(-\infty)$. L'Hospital's Rule won't work on products, it only works on quotients. However, we can turn this into a fraction if we rewrite things a little.

$$\lim_{x \to 0^+} x \ln x = \lim_{x \to 0^+} \frac{\ln x}{{}^1/_x}$$

The function is the same, just rewritten, and the limit is now in the form $-\infty/\infty$ and we can now use L'Hospital's Rule.

$$\lim_{x \to 0^+} x \ln x = \lim_{x \to 0^+} \frac{\ln x}{{}^1/_x} = \lim_{x \to 0^+} \frac{{}^1/_x}{-{}^1/_{x^2}}$$

Now, this is a mess, but it cleans up nicely.

$$\lim_{x \to 0^+} x \ln x = \lim_{x \to 0^+} \frac{{}^1/_x}{-{}^1/_{x^2}} = \lim_{x \to 0^+} (-x) = 0$$

Practice:

1) Find the limit $\lim_{x \to \infty} \frac{x^2}{2^x}$.

2) Calculate the limit $\lim_{x \to 2} \left(\frac{4}{x^2-4} - \frac{1}{x-2} \right)$.

Answer Key:

Once you have completed the problems, check your answers here.

1) *Solution.*

Using L'Hopital's rule, we can write

$$\lim_{x\to\infty}\frac{x^2}{2^x} = \left[\frac{\infty}{\infty}\right] = \lim_{x\to\infty}\frac{(x^2)'}{(2^x)'} = \lim_{x\to\infty}\frac{2x}{2^x\ln 2} = \frac{2}{\ln 2}\lim_{x\to\infty}\frac{x}{2^x} = \left[\frac{\infty}{\infty}\right]$$

$$= \frac{2}{\ln 2}\lim_{x\to\infty}\frac{(x)'}{(2^x)'} = \frac{2}{\ln 2}\lim_{x\to\infty}\frac{1}{2^x\ln 2} = \frac{2}{(\ln 2)^2}\lim_{x\to\infty}\frac{1}{2^x} = \frac{2}{(\ln 2)^2}\cdot 0 = 0.$$

2) *Solution.*

Here we deal with an inderminate form of type $\infty - \infty$. After simple transformations, we have

$$\lim_{x\to 2}\left(\frac{4}{x^2-4} - \frac{1}{x-2}\right) = \lim_{x\to 2}\frac{4-(x+2)}{x^2-4} = \lim_{x\to 2}\frac{2-x}{x^2-4} = \left[\frac{0}{0}\right] = \lim_{x\to 2}\frac{(2-x)'}{(x^2-4)'}$$

$$= \lim_{x\to 2}\left(\frac{-1}{2x}\right) = -\frac{1}{4}.$$

Additional Practice:

In your Calculus book read through section 8.7 and complete problems 5, 7, 9, 11, 15, 19, 23, and 33 on page 574

Interactive Practice

More Interactive Practice

Extra Practice with Answers